Discovering Math
Concepts in Advanced Algebra
Teacher's Guide

Grade Level: 10–12	**Curriculum Focus:** Mathematics	**Lesson Duration:** Three class periods

Program Description

***Concepts in Advanced Algebra*—**Calculate odds, measure earthquakes, ride some fantastic roller coasters, investigate the making and breaking of codes, and see how math helps protect us everyday, and explore the ancient Chinese mathematical system.

Onscreen Questions

- Why is it practical for the Richter scale to use logarithmic measurements?
- How is math used every day to protect us?
- In what ways are Western and ancient Chinese mathematical systems similar?
- How are the mathematical achievements of ancient China different from those of the ancient Greeks and Egyptians?

Lesson Plan

Student Objectives

- Discover Chinese achievements in mathematics.
- Convert Chinese numbers and decimal numbers.
- Solve excess and deficiency problems.
- Solve 3 × 3 systems of equations.

Materials

- *Discovering Math: Concepts in Advanced Algebra* video
- Computer with Internet access
- Print resources about the history of Chinese mathematics

Procedures

1. Have students research *The Nine Chapters on the Mathematical Art* and create an illustrated table of contents for each chapter using print and Web resources. The following Web sites are a good starting point:
   - The Nine Chapters on the Mathematical Art
     http://en.wikipedia.org/wiki/The_Nine_Chapters_on_the_Mathematical_Art
   - Nine chapters
     http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Nine_chapters.html
   - The Nine Chapters on the Mathematical Art
     http://www.math.sfu.ca/histmath/China/1stCenturyAD/NineChapIntro.html

2. Have students research Chinese contributions to mathematics using print and Web resources. Useful information is in the following Web sites:
   - History of Mathematics: China
     http://aleph0.clarku.edu/~djoyce/mathhist/china.html
   - Chinese overview
     http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/Chinese_overview.html#s31
3. When students have completed their research, ask them to summarize their findings in a one-page report.
4. Ask students to choose a partner to share their report and answer any questions. Then have each student summarize the partner's report for the class, making sure to include at least three interesting facts.
5. Show students examples of converting Chinese numbers to decimal numbers and converting decimal numbers to Chinese numbers. Be sure to include numbers that contain the digit 0. Allow enough time for practice.
6. Provide students with an example of an excess and deficiency problem:

   Suppose a group of people is purchasing merchandise. If each person contributes six coins, there is an excess of two coins. If each person contributes five coins, there is a deficit of one coin. How many people are in the group? What is the price of the merchandise?
   Solution: Solve the problem by substitution. The price of the merchandise is 16 coins and the number of people in the group is three.
7. Use the Chinese method to solve the above problem: If each person contributes a_1, then there is b_1 either in excess or deficit. If each person contributes a_2, then there is b_2 either in excess or deficit. The price of the merchandise is $a_1b_2 + b_1a_2$. The number of people is $b_1 + b_2$. Each person should pay $a_1b_2 + b_1a_2$ divided by $b_1 + b_2$.

8. Give students an example of problem involving a 3 × 3 system of equations:
   There are three different types of corn: corn x, corn y, and corn z.

   3 bundles of corn x plus 2 bundles of corn y plus 1 bundle of corn z equal 22 pounds of corn.

   $3x + 2y + z = 22$

   4 bundles of corn x plus 6 bundles of corn y plus 2 bundles of corn z equal 48 pounds of corn.
   $4x + 6y + 2z = 48$

   1 bundle of corn x plus 2 bundles of corn y plus 3 bundles of corn z equal 34 pounds of corn.

   $x + 2y + 3z = 34$

   Set up and solve the problem using the Chinese method:

1	4	3	1	12	3	1	0	3	3	0	3	C1 = column 1
2	6	2	2	18	2	2	10	2	6	10	2	C2 = column 2
3	2	1	3	6	1	3	2	1	9	2	1	C3 = column 3
34	48	22	34	144	22	34	56	22	102	56	22	
				3C2			C2 – 4C3			3C1		

0	0	3	0	0	3	0	0	3
4	10	2	40	10	2	0	10	2
8	2	1	80	2	1	72	2	1
80	56	22	800	56	22	576	56	22
C1 – C3				10C1			C1 – 4C2	

   From column 1, $72z = 576$. $z = 8$

   From column 2, $0 + 10y + 2(8) = 56$. $y = 4$

   From column 3, $3x + 2(4) + 8 = 22$. $x = 2$

9. Give students an example of a problem involving a 3 × 3 system of equations that uses negative numbers to reach the solution.

   Solve the system of equations:

   $3x + y = 1$

   $y + z = 1$

   $2x + 2z = 1$

Set up and solve the problem using the Chinese method.

2	0	3	6	0	3	0	0	3	0	0	3	0	0	3
0	1	1	0	1	1	–2	1	1	–2	2	1	0	2	1

2	1	0		6	1	0		6	1	0		6	2	0		8	2	0
1	1	1		3	1	1		1	1	1		1	2	1		3	2	1
					3C1				C1 – 2C3				2C2				C1 + C2	

From column 1, $8z = 3.\ z = \frac{3}{8}$

From column 2, $2y + 2\left(\frac{3}{8}\right) = 2.\ y = \frac{5}{8}$

From column 3, $3x + \frac{5}{8} = 1.\ x = \frac{1}{8}$

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students were highly engaged in class discussions; produced complete reports, including all of the requested information; clearly demonstrated the ability to convert between Chinese and decimal numbers, the ability to solve excess and deficiency problems, and the ability to solve 3 × 3 systems of equations.
- **2 points:** Students participated in class discussions; produced an adequate report, including most of the requested information; satisfactorily demonstrated the ability to convert between Chinese and decimal numbers, the ability to solve excess and deficiency problems, and the ability to solve 3 × 3 systems of equations.
- **1 point:** Students participated minimally in class discussions; created an incomplete report with little or none of the requested information; were not able to convert between Chinese and decimal numbers, solve excess and deficiency problems, or solve 3 × 3 systems of equations.

Vocabulary

Chinese counting board

Definition: A checkerboard with rows and columns where numbers were represented using small rods made from bamboo or ivory: A number was formed in a row with the units placed in the rightmost column, the tens in the next column to the left, the hundreds in the next column to the left, and so on.

Context: The most significant attribute of the counting board was that it presented a place-value number system.

column of a rectangular array

Definition: One of the vertical lines of elements in a rectangular array

Context: Using a counting board, the Chinese would solve a 3 × 3 system of equations by setting up the coefficients of each equation in side-by-side columns, creating a rectangular array.

rectangular array

Definition: An arrangement of mathematical elements into rows and columns

Context: Chapter 8 of *The Nine Chapters on the Mathematical Art* is "Rectangular Arrays," offering a method for solving systems of simultaneous linear equations.

row of a rectangular array

Definition: One of the horizontal lines of elements in a rectangular array

Context: To solve a 3 × 3 system of equations, the Chinese set up the coefficients of each equation into a rectangular array of three rows and three columns.

3 × 3 system of linear equations

Definition: A collection of three linear equations with three unknowns

Context: Chapter 8 of *The Nine Chapters on the Mathematical Art* solves 3 × 3 systems of equations.

2 × 2 system of linear equations

Definition: A collection of two linear equations with two unknowns

Context: Excess and deficiency problems can be represented by 2 × 2 systems of equations.

Academic Standards

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics provides guidelines for teaching mathematics in grades K–12 to promote mathematical literacy. To view the standards, visit this Web site: http://standards.nctm.org/document/chapter3/index.htm

This lesson plan addresses the following thematic standards:

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K–12 Education addresses 14 content areas. To view the standards and benchmarks, visit http://www.mcrel.org/compendium/browse.asp .

This lesson plan addresses the following national standards:

- Mathematics: Understands counting procedures and reasoning (e.g., use of the counting principles to find the number of ways of arranging objects in a set, the use of permutations and combinations to solve counting problems); Understands basic applications of and operations on matrices; Understands and applies basic and advanced properties of functions and algebra
- Science: Physical Science: Understands the structure and properties of matter; Understands the sources and properties of energy
- World History: Understands Chinese achievements in mathematics

- Historical Understanding: Understands evidence of social and cultural development of Chinese civilization

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Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- http://school.discovery.com/teachingtools/teachingtools.html

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DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How To Use the DVD

The DVD starting screen has the following options:

***Play Video*—** This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index—Here the video is divided into sections indicated by video thumbnail icons; brief descriptions are noted for each one. Watching all parts in sequence is similar to watching the video from start to finish. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

***Curriculum Units*—** These are specially edited video segments pulled from different sections of the video (see below). These nonlinear segments align with key ideas in the unit of instruction. They include onscreen pre- and post-viewing questions, reproduced below in this Teacher's Guide. Total running times for these segments are noted. To play a particular segment, press Enter on the TV remote or click once on the Curriculum Unit title on a computer.

***Standards Link*—** Selecting this option displays a single screen that lists the national academic standards the video addresses.

***Teacher Resources*—** This screen gives the technical support number and Web site address.

Video Index

I. Permutations and Combinations (5 min.)

Learn how permutations and combinations can help you count all the possible outcomes at sporting events.

II. Logarithms (6 min.)

An earthquake's magnitude is a logarithmic measure of its size. Learn how to calculate such magnitude using the Richter scale and how to compare the intensity of two earthquakes using exponential functions.

III. Cubic Functions (5 min.)

The path of some roller coasters can be modeled by cubic functions. Learn how to approximate the slope of the descent of a roller coaster using these cubic functions.

IV. Matrix Multiplication (6min.)

Learn what a matrix is and how to add, subtract, and multiply them. Discover how to use matrix multiplication to code and decode messages.

V. Math and Chinese Civilization (26 min.)

The Chinese developed their mathematical system with no outside influence. Learn how the Chinese write their numbers using a counting board, as well as the ingenious methods they had for solving practical problems.

Curriculum Units

1. Permutations and Combinations

Pre-viewing question
Q: What is an example of a situation in which order counts? What is an example of a situation in which order does not count?
A: Answers will vary.
Post-viewing question
Q: What is the formula for the permutations of size k that can be taken from a set of size n? What is the formula for the combinations of size k that can be taken from a set of size n?

A: The formula for the permutations of size k that can be taken from a set of size n is ${}_nP_k = \frac{n!}{(n-k)!}$.

The formula for the combinations of size k that can be taken from a set of size n is ${}_nC_k = \frac{n!}{k!(n-k)!}$.

2. Logarithms and Earthquake Magnitude

Pre-viewing question
Q: What causes earthquakes?
A: Answers will vary.
Post-viewing question
Q: If an earthquake located 150 kilometers away has a seismographic reading with amplitude of 35 millimeters, what would the magnitude on the Richter scale be?
A: Use the formula $M = \log A - D$, where A = amplitude (mm) and D = distance correction factor. From Richter's chart, the distance-correction factor D is –3.29.

$$\begin{aligned} M &= \log A - D \\ &= \log 35 - D \\ &= 1.544 - D \\ &= 1.544 - (-3.29) \\ &= 4.8 \end{aligned}$$

3. Exponential Functions and Earthquake Intensities

Pre-viewing question
Q: What is the worst natural disaster that has occurred in your lifetime?
A: Answers will vary.
Post-viewing question
Q: How much larger in intensity is a 9.0 earthquake than an 8.5 earthquake?
A: $M = \log A$
$A = 10^M$

$$\begin{aligned} 9.0 - 8.5 &= 0.5 \\ A &= 10^M \\ &= 10^{0.5} \\ &\approx 3.2 \\ &\approx 3.2 \end{aligned}$$

4. Cubic Functions

Pre-viewing question
Q: What is an example of a cubic polynomial?
A: Answers will vary.
Post-viewing question
Q: How was the approximate slope of the descent of the roller coaster path

$f(x) = x^3 - x^2 - 4x + 4$ found in the video?
A: The y-intercept was found to be (0, 4). One of the x-intercepts was found to be (1, 0). The slope of the line through these 2 points approximates the slope of the descent. $m = \frac{0-4}{2-0} = -2$

5. Matrix Arithmetic

Pre-viewing question

Q: What are some examples of data that is presented in rectangular arrays of numbers?
A: Answers will vary.
Post-viewing question
Q: If $A=\begin{bmatrix}2 & 1\\ -1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}0 & 3\\ 2 & 1\end{bmatrix}$, what is $A + B$, $A - B$, and $A \times B$?
A: $A + B = \begin{bmatrix}2 & 4\\ 1 & 1\end{bmatrix}$ $A - B = \begin{bmatrix}2 & -2\\ -3 & -1\end{bmatrix}$, and $A \times B = \begin{bmatrix}2 & 7\\ 0 & -3\end{bmatrix}$

6. Matrix Multiplications Code and Decode Messages

Pre-viewing question
Q: Why would someone want to code a message?
A: Answers will vary.
Post-viewing question
Q: How does the Hill cipher work?
A: Answers will vary.

7. Math and Chinese Civilization

Pre-viewing question
Q: Where is China?
A: China is located in East Asia, bordering the East China Sea, Korea Bay, Yellow Sea, and South China Sea, between North Korea and Vietnam.
Post-viewing question
Q: What is unique about the development of the Chinese mathematical system?
A: Answers will vary. The Chinese developed their mathematical system with no outside influence.

8. Chinese Numbers

Pre-viewing question
Q: Which number systems have numerals different from those in our decimal number system?
A: Answers will vary.
Post-viewing question
Q: How did the Chinese use the counting board to represent numbers?
A: Answers will vary.

9. Solving an Excess and Deficiency Problem

Pre-viewing question
Q: How would you solve a 2×2 system of equations?
A: Answers will vary.

Post-viewing question
Q: If each person contributes 9, there is an excess of 4. If each person contributes 6, there is a deficit

of 2. How many people are in the group and how much is the price of the merchandise?
A: Let n be the number of people and p be the price of the merchandise, then

$p = 9n - 4$
$p = 6n + 2$
$9n - 4 = 6n + 2$
$3n = 6$
$n = 2$
$p = 9n - 4$
$p = 9(2) - 4$
$p = 18 - 4$
$p = 14$

The price of the item is 14 and the number of people is 2.

10. Chinese Method for Solving an Excess and Deficiency Problem

Pre-viewing question
Q: Which type of equations do you use a formula to solve?
A: Answers will vary.
Post-viewing question
Q: How do the Chinese solve excess and deficiency problems?
A: Answers will vary

11. Chinese Method for Solving a 3 × 3 System of Equations

Pre-viewing question
Q: How would you solve a 3 × 3 system of equations?
A: Answers will vary.
Post-viewing question
Q: How do the Chinese solve a 3 × 3 system of equations?
A: Answers will vary.

12. Chinese Mathematics and Negative Numbers

Pre-viewing question
Q: What is the purpose of negative numbers?
A: Answers will vary.
Post-viewing question
Q: How did the Chinese use negative numbers?
A: Answers will vary.